



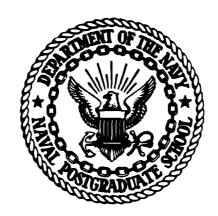
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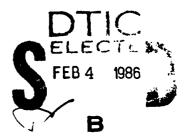
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# NAVAL POSTGRADUATE SCHOOL Monterey, California





SIMULATION AND PERFORMANCE OF BRUSHLESS DC MOTOR ACTUATORS

Alex Gerba Jr.

December 1985

Progress Report for Period

October 1984 - September 1985

Approved for Public Release; Distribution Unlimited Prepared for: Naval Weapons Center, Code 3275
China Lake, California 93555

# NAVAL POSTGRADUATE SCHOOL Monterey, California

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IN SUPPORT OF THE PROGRAM "ADVANCED MISSILE CONTROL DEVICES"

of the

Naval Weapons Center China Lake, California

December 1985

For the period October 1984 - September 1985

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## SIMULATION AND PERFORMANCE OF BRUSHLESS DC MOTOR ACTUATORS

#### SUMMARY

The simulation model for a Brushless D.C. Motor and the associated commutation power conditioner transistor model are presented. The necessary conditions for maximum power output while operating at steady-state speed and sinusoidally distributed air-gap flux are developed.

Comparisons of simulated model with the measured performance of a typical motor are done both on time response waveforms and on average performance characteristics. These preliminary results indicate good agreement. Plans for model improvement and testing of a motor-driven positioning device for model evaluation are outlined.

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#### - LIST OF SYMBOLS

- $B_m$  Viscous friction coefficient of the motor (oz-in/rad per sec)
- B<sub>1</sub> Viscous friction coefficient of the load (oz-in/rad per sec)
- $E_{\mathsf{BA}}$  Back EMF of phase A (volts)
- EBB Back EMF of phase B (volts)
- EBC Back EMF of phase C (volts)
- IA Phase A current (amperes)
- IB Phase B current (amperes)
- I<sub>C</sub> Phase C current (amperes)
- I<sub>M</sub> Power supply current (amperes)
- $J_M$  Moment of inertia of motor shaft (oz-in-sec<sup>2</sup>)
- $J_1$  Moment of inertia of load shaft (oz-in-sec<sup>2</sup>)
- kb Back EMF constant (volts/rad per sec)
- kt Torque constant (oz-in/ampere)
- L Inductance of the stator winding (Henrys)
- IAB, ICA, IBC Loop currents (amperes)
- kadi Air-gap flux adjustment factor
- $k_{\rm W1}$  Magnitude of speed versus torque curve slope (rad per sec/oz-in)
- Ra Resistance of the stator windings (ohms)
- R<sub>S</sub> Power supply interval resistance (ohms)
- $\phi_A$ ,  $\phi_B$ ,  $\phi_C$  Per phase air-gap flux (webers)
- $R_{\text{n}}$  'On' resistance of the power transistor (Ohms)
- T<sub>m</sub> Motor restraining torque (oz-in)
- T<sub>1</sub> Load restraining torque (oz-in)
- $T_A$ ,  $T_B$ ,  $T_C$  Per phase developed torque (oz-in)
- Wm Speed of the motor shaft (rad/sec)

GR Gear ratio

8 Angular displacement (radians)

 $V_A$ ,  $V_B$ ,  $V_C$  Phase terminal voltage (volts)

V<sub>S</sub> Power supply terminal voltage (volts)

RPSA, RPSB, RPSC Position sensors logic signals

PWM Pulse width modulation

#### INTRODUCTION

Recent improvements in rare-earth magnetic materials for use in brushless dc motors have allowed reconsideration of electro-mechanical actuator systems for applications requiring very high ratios of torque-to-inertia. The investigation discussed herein has been concerned with characterizing mathematically the dynamical features of a missile fin actuation system, from the input to the brushless dc motor to the output shaft of the mechanical actuator. The physical model is based upon an existing prototype actuator currently under evaluation at the Naval Weapons Center, China Lake, California.

In general, brushless do motors produce torque through the interaction of a magnetic field generated by a permanent magnet rotor and a do generated magnetic field in the stator. The rotating permanent magnet eliminates the rotating armature and the mechanical wear normally associated with brushes. These motors fall in the class of <u>Permanent Magnet Motors</u> and enjoy certain advantages over wound-field types such as:

"...Linear torque-speed characteristics, high stall (accelerating) torque, no need for electric power to generate the magnetic flux and a smaller frame and lighter motor for a given output power" [1].

Additionally, the brushless dc motor is characterized by:

"...controllability over a wide range of speeds, capable of rapid acceleration and deceleration, convenient control of shaft speed and position, no mechanical wear problem due to commutation and better heat dissipation arrangement" [1].

The fundamental requirement of an electro-mechanical actuator control system is to provide torque to an output shaft, sense the position of the shaft and adjust the torque to balance the load when the desired position is reached. This must be accomplished with a minimum of frictional resistance and delays associated with the inertia of the mechanical components. Effects of viscous, static and coulomb friction, together with the torque required to accelerate the mechanical components of the system, lead to a reduction in

torque available at the output shaft and an associated reduction in system performance.

One approach to the analysis of the electro-mechanical actuator system has been to divide the system into two sequential problem areas. The first deals with the dynamic analysis of the brushless dc motor and development of the transfer function necessary to duplicate actual steady-state and transient performance. The second area deals with modeling the mechanical system elements, taking as input the dc motor shaft angular acceleration predicted by the motor analysis. The mechanical system must be modeled considering the effects of friction and inertia and translating the rotational motion of the brushless dc motor shaft to the output shaft of the actuator for application to missile maneuvering control. The results obtained from a study that has placed primary emphasis upon the latter problem area - the modeling of the mechanical drive-train leading to the fin shaft was presented in Ref. 3 and 4.

This report deals with the first problem area and presents an overview of the work done by MacMillan [4]. In Ref. 4, MacMillan developed a model for the output circuitry of a transistorized power conditioner that provided the required motor commutation. The model developed in Ref. 4 used an ideal (zero impedance) power supply. This report extends the model to include internal power supply resistance and also identifies areas for further improvement in the system modeling.

The next section of this report presents a brief description of the system followed by a development of the model and an analysis of the simulated motor performance. Results of the simulated system are then compared to the measured response from a typical commercially available motor and recommendations for further improvements in the model are outlined.

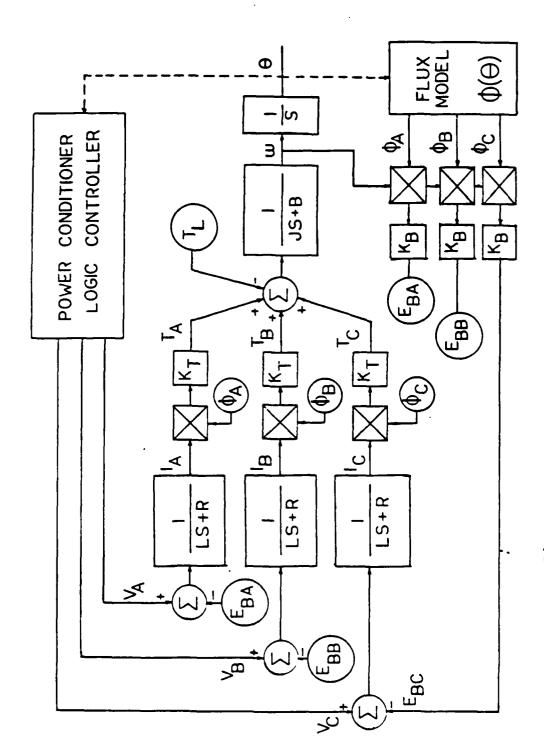
#### - SYSTEM DESCRIPTION

**GENERAL** 

The system is viewed as a position control device to maintain an output angle under an applied hinge moment due to aerodynamic forces on a fin or aileron. The motor is a permanent magnet dc motor with feedback in the form of back emf proportional to the angular velocity of the motor. The block diagram of the dc motor and power conditioner is shown in Fig. 1 [4]. The mechanical actuator and drive train, as currently envisioned, introduce various inertial and damping loads together with an aerodynamic force and its associated fin hinge moment that must be overcome to produce output motion. An operational block diagram of the load torque is shown in Fig. 2 where the hinge moment and motor shaft angular acceleration are viewed as inputs to the drive train [2]. Figure 3 is a schematic of the drive train which, as presently constituted, includes the motor shaft (leadscrew), ball screw assembly, and the crank which is keyed to the output (fin) shaft. Inertial loads are considered individually within three major subdivisions of the actuator; the output shaft to crank, crank to ball screw, and ball screw to leadscrew.

A detailed analysis and development of the mechanical model with the associated assumptions are contained in Reference 2. As stated in the introduction, this report reviews the development of the motor model reported in Reference 4. For convenience in the development of the model, MacMillan considered the load seen by the motor to be caused by the inertia and kinetic (viscous) friction of the motor shaft as described by Thomas in Reference 5.

The schematic diagram of the power supply, power conditioner and motor-load is shown in Figure 4 where it is noted that the power conditioner logic and driver circuits are modeled as ideal on-off devices with zero time delay.



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Pigure | Motor and Power Conditioner Simulation Diagram

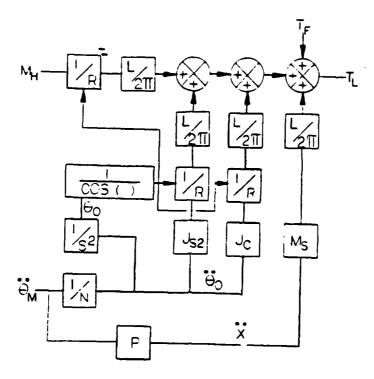


Figure 2 Load Torque Block Diagram

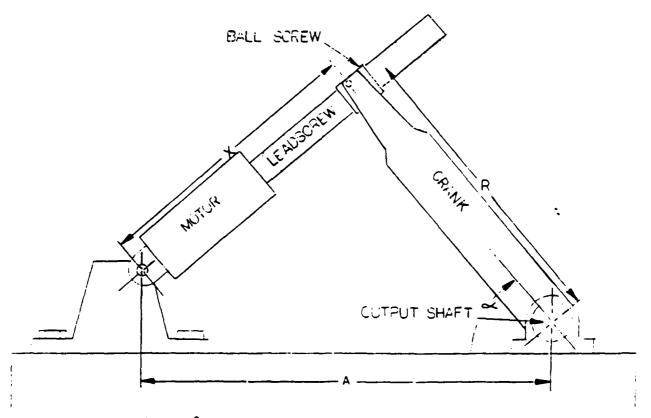
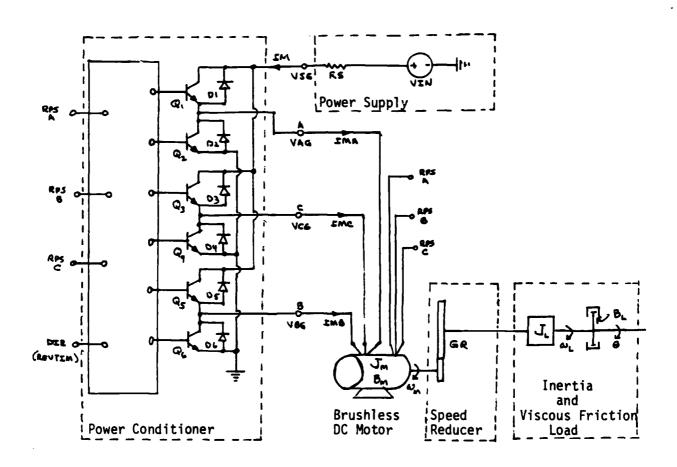


Figure 3 Sketch of Actuator Arrangement



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Figure 4 Schematic Diagram of Power Supply, Power Conditioner and Motor with Inertia and Viscous Friction Load.

Additionally, MacMillan considered the power supply to be ideal (zero interval resistance) to further reduce the complexity and simplify the development of the power conditioner model. However, in this report, the model used for the power supply includes the internal resistance, R<sub>S</sub>. The effect of including R<sub>S</sub> in the requirements and performance of the system model is presented in the section on Analysis of Simulated Motor Performance. For additional information on the development of the D.C. motor model refer to Reference 4.

#### DEVELOPMENT OF SYSTEM MODEL

The basic simulation by Thomas used a single power supply and superimposed the phase currents to produce the motor torque. Motor drive was then realized by multiplying averaged armature current by a torque factor.

In the model developed by MacMillan, the supply is considered to be a split supply of equal voltages. Armature current is assigned a positive sign if it flows in the positive direction (i.e. into the motor). The use of a split power supply allowed MacMillan to apply circuit reduction techniques. Because of the symmetry that resulted from the split power supply approach, the complex 3-phase bridge-type circuit simplified into a two-window network. The development of the reduced circuit begins with the definitions of loop currents as shown in Figure 5 where the power supply voltage of 2V is represented as V+ and V-. The simulated model equivalent circuit is shown in Fig. 6 with node N defined as the mid-point of the power supply and node 0 to be the mid-point of the motor windings and neither of these nodes are considered to be at ground potential (the ground is identified as the Reference node). Note that the power supply, VIN = VIF - VIB = 2V and that all the network variables are defined using the CSMP model variable definitions [6]. Given the assumption that the network is balanced, that is,

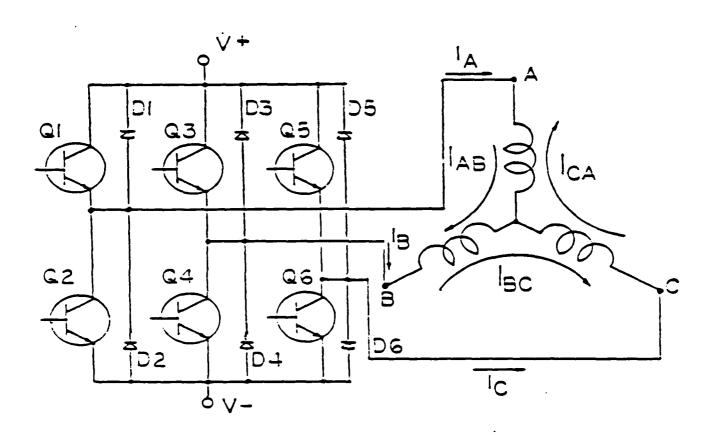


Figure 5 Motor and Commutating Transistors

all transistors, diodes and field windings are approximated as being alike in characteristics, it is possible to apply the circuit reduction technique of Thevenin and obtain the simple 2-window network shown in Figure 7 [4]. In Figure 7, the Thevenin equivalent voltages and currents are defined in terms of the CSMP model variables.

The back emf voltages of each phase (VEMFA, VEMFB and VEMFC) are developed and presented in Reference 4 in which it is shown that these voltages are summed 2 at a time in proper sequence to compute the loop currents and resulting phase torque. The total developed torque is then the sum of the 2-active winding phase torques over the proper 60 degree of mechanical angle. Figure 8 shows the back emf voltages generated across 2 windings taken 2 at a time. The 60 degree of mechanical angle that produces torque is shown by the logic unit output levels of the position transducer RPSA, RPSB and RPSC in Figure 8.

Reference 4 presents a detailed development of the power conditioner model that includes the assumptions for the switching transistor dynamics used in the power conditioner model as well as the development of the harmonic air-gap flux used in the motor model.

#### ANALYSIS OF SIMULATED MOTOR PERFORMANCE

The proper evaluation of a system model requires, quite naturally, a comparison of the actual system response for the same type of input function used for the model. For the brushless D.C. motor, it is assumed that measurements can be obtained for the following voltages and currents:

- Power supply terminal voltage; V<sub>SR</sub>
- Voltage across two of the three windings;  $V_{ab}$ ,  $V_{bc}$ , and  $V_{ca}$

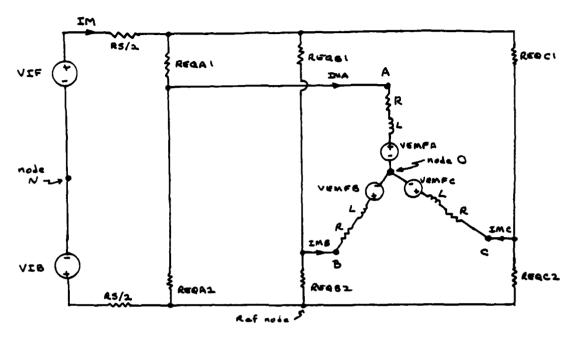


Figure 5 Split Power Supply Bridge-Type Circuit Model

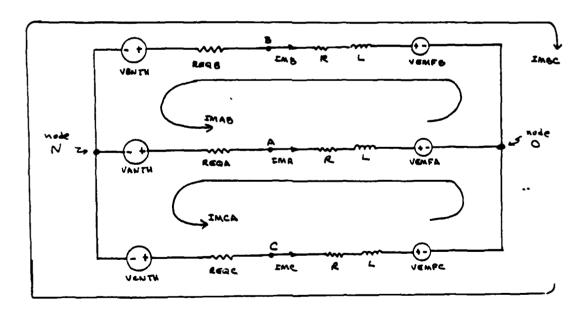
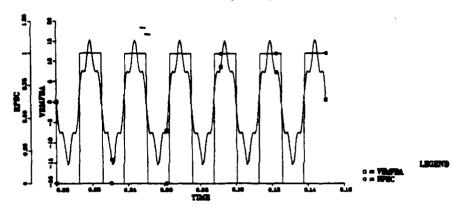
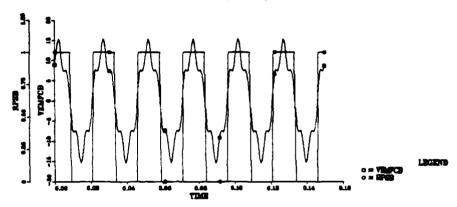


Figure 6 Two-Window Equivalent Circuit of the Balanced Bridge Network

#### BACK BAT ACROSS B-A WINDONGS (1200 RPM) - VOLTS



#### BACK EMP ACROSS C-B WEITHINGS (1200 RPM) - VOLUS



#### BACK EMF ACROSS A-C WINDLINGS (1200 RPM) - VOLTS

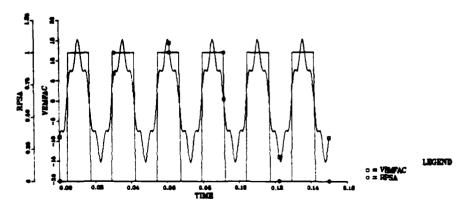


Figure 8 Back EMF and Position Sensor Output with Motor Driven at 1200 RPM (CCW)

- Voltage from one winding terminal to ground;  $\text{V}_{\text{a}}\text{, }\text{V}_{\text{b}}$  and  $\text{V}_{\text{C}}$
- Power supply current; Im
- Phase current; Ima, Imb, Imc

Before taking data from the simulated model, it is necessary to set all constants and parameters to the nominal values given by the supplier of the motor. Care must be taken to properly interpret the values given by the manufacturer.

In particular, the value of the Back EMF coefficient,  $k_{\rm b}$ , needs special attention. If the motor data sheet indicates that this coefficient is obtained by measurement of peak to peak voltage across two windings, then in the model that uses distributed sinusoidal air-gap flux, an adjustment factor,  $k_{\rm adj}$  is required to insure that the Back EMF values are in agreement. It follows that the torque coefficient,  $k_{\rm t}$ , must also be adjusted by this same scale factor [7]. The value of  $k_{\rm adj}$  can be computed from the measured no-load data for the motor current, speed and applied voltage by using the average characteristics balance equation,

$$i_m = (v_{sg} - k_b w_{n1})/(R_a + R_q)$$

where  $k_b = k_{bm} k_{adj}$ 

kbm = manufacturer supplied Back EMF constant.

Thus the adjustment factor becomes

$$k_{adj} = [v_{sg} - i_m(R_a + R_q)]/(k_{bm}w_{nl})$$

For example, given a no-load speed of 3060 RPM with a terminal voltage of 30 volts and a no-load current of 0.30 amperes,  $k_{adj} = 0.825$  (given a total series resistance of 1.47 ohms). The value of  $k_{adj}$  must of course be less than unity, otherwise the motor current would be negative in value which implies that generator rather than motor action is taking place.

Before an attempt is made to close the loop on the motor-load to form a positioning device, it is important to operate the system as a velocity device and obtain measurement of the motor average performance characteristics. Typical motor average performance curves are speed versus torque, motor current versus torque and output power versus torque from no-load torque to near stall torque conditions. Figure 9 shows these three curves for the model with torque given in oz-in units. Note in particular the straight line characteristics for motor speed and current and that the power output is of quadratic form. The condition of constant input voltage is used when gathering data for these curves, and it follows that if speed is a straight-line versus torque, then current also has a straight-line when plotted against torque. This can be shown as follows:

$$w_m = w_{n1} - k_{w1}T$$

where  $w_m$  = motor speed

 $w_{n1} = no-load speed$ 

 $k_{wl}$  = magnitude of slope of the speed vs torque curve

T = load torque

The current, using average values, can be written as

$$i_m = (v_{sg} - k_b w_m)/(R_a + R_q)$$

where  $v_{sg}$  = power supply terminal voltage

kb = Back EMF coefficient

Ra = winding resistance

R<sub>q</sub> = transistor "ON" resistance

then by substitution,

$$i_m = [(v_{sg} - k_b w_{n1})/(R_a + R_q)] + [(k_b k_{w1})/(R_a + R_q)]T$$

where the first term on the right hand side of the equation represents the no-load current.

## OUTPUT POWER (WATTS) VS LOAD TORQUE (OZ-IN)

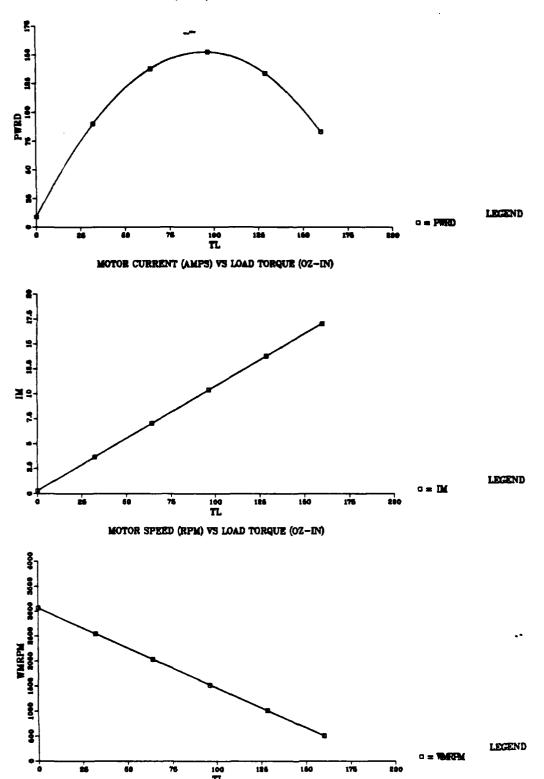


Figure 9 Average Performance Characteristics of the Simulated Model

The power output versus torque curve in Figure 9 reaches a peak value between no-load torque and stall torque. The analytical development for this quadratic curve is presented in Appendix A where it is shown that the condition for peak power output,  $P_0$ , is that the torque must be of value

$$T = (2k_b w_{nl} - v_{sg})/(2k_b k_{wl})$$

to produce maximum power output

$$P_o = k_t v_{sg}^2 / (4R_s k_b)$$

Since the ratio  $k_t/k_b$  is a constant, increasing peak power output depends upon decreasing the power supply interval resistance,  $R_s$ , or increasing the power supply voltage,  $v_{sg}$  as one would logically expect. It must be understood that the assumption used in the above analysis was that the airgap flux was a constant average value. For the actual motor the air-gap flux is distributed sinusoidally and current in the windings produces a field that may cause distortion in the air-gap flux. At large current values that occur at and above peak power output, the field distortion may result in an increase or a reduction of output power for a given load torque.

Another motor coefficient to consider carefully is the no-load viscous friction constant,  $B_m$ . The value of  $B_m$  is often not included in the motor data sheet and the probable reason is that its value will depend somewhat on the manner in which the motor is attached to the system load.  $B_m$  can be calculated from given no-load data as follows:

$$B_m = (k_t k_{adj} i_m)/w_{nl}$$

where  $i_{m}$  is the no-load current.

#### RESULTS AND CONCLUSIONS

The preliminary results indicate that the balanced bridge circuit approach used in the development of the Brushless DC motor model produces good

agreement with measured motor characteristics as indicated below. Further evaluations of the model as well as improvements and additions to the model will be conducted in the near future using data currently being gathered from a prototype of a fin positioning actuator.

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A comparison of model and motor produced Back EMF voltage is shown in Figure 10. The upper curve is the simulated motor waveform and the lower curve is the measured Back EMF. The model adjustment factor,  $k_{adj}$  was set to 0.63 value by trial and error until peak to peak voltages were in agreement. Another verification of the model is obtained by comparison of Figure 11 and Figure 8. Both figures show waveforms of Back EMF across 2 windings taken 2 at a time and also show the timing waveforms for the Position Sensor Devices (RPSA, RPSB and RPSC) for counter clockwise rotation. The waveforms agree in both phasing and in form.

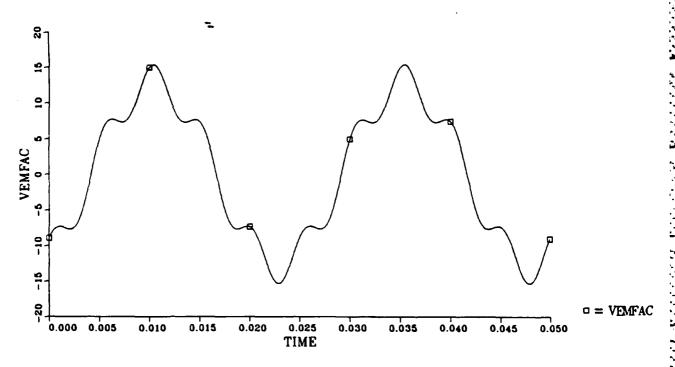
The steady-state performance curves of a typical motor are given in Figure 12 where the motor load torque is given in 1b in units. These curves were produced with a constant terminal voltage of 30 volts and agree in form with the curves of the model as given in Figure 9. Peak power output occurs at load torque of approximately 95 oz-in for both motor and model.

Additional validation of the model is shown in Figure 13 where the current in Phase C for the model (upper curve) is in close agreement both in form and in phase with the same current for the motor (lower curve). Additional model results are shown in Appendix B where typical input voltage step response waveforms are presented.

#### FURTHER RESEARCH

Improvements in the model are required with regard to the triggering on of the diodes used to protect the transistors from excessive reverse currents

# BACK EMF VOLTAGE ACROSS A-C WINDINGS (1200RPM)-VOLTS



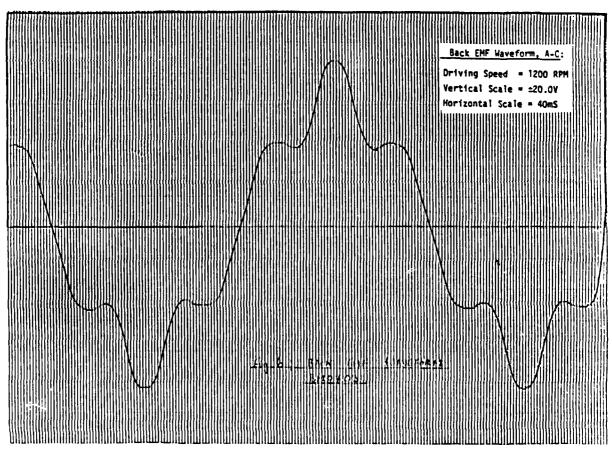


Figure 10 Back EMF Voltage: Top Curve-Model Output, \*\* Bottom Curve-Typical Motor Output

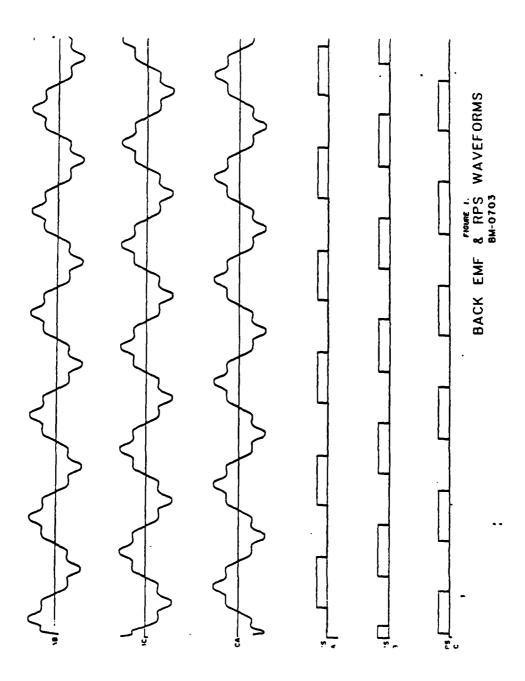


Figure 11 Back EMF and Position Sensor Output Waveform for a Typical Motor

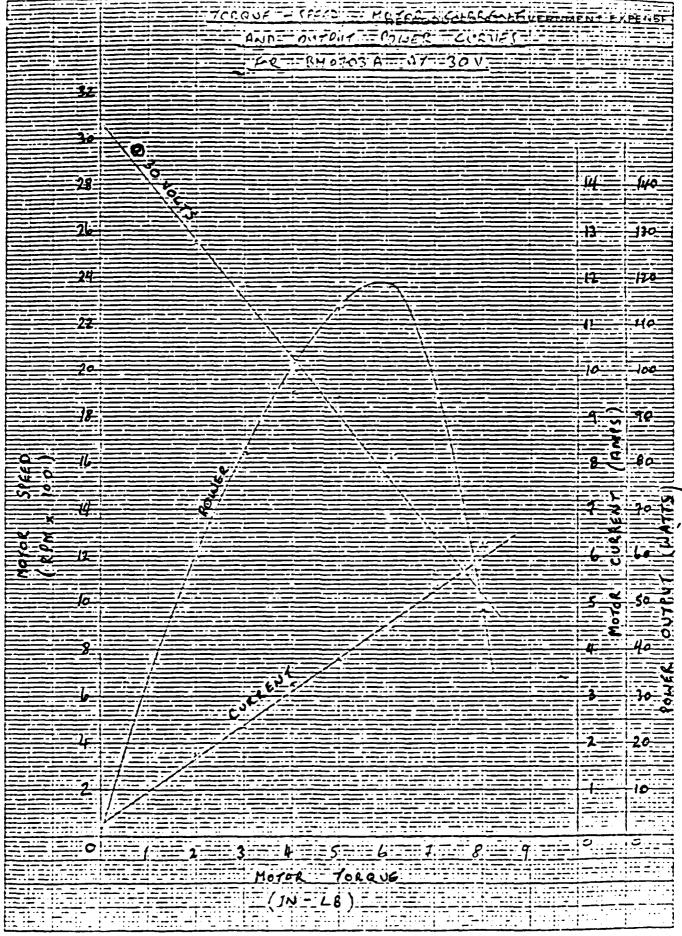
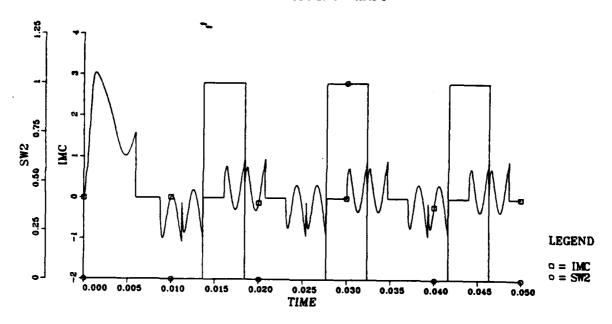


Figure 12 Typical Measured Motor Performance

#### PHASE C CURRENT - AMPS



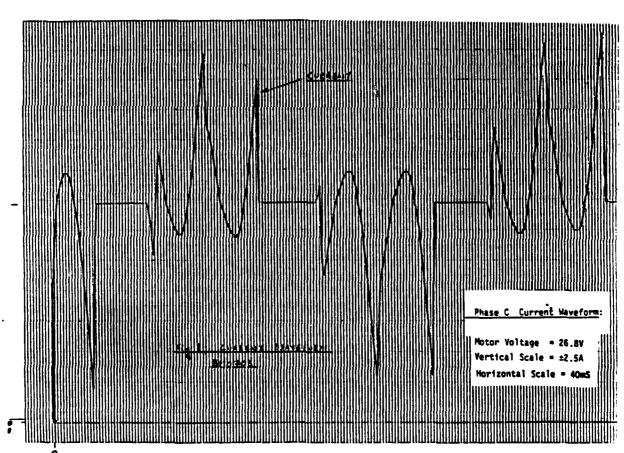


Figure 13 Phace C Current: Top Curve-Simulated Model, Lower Curve-Typical Motor

during the switch off time for the transistors. MacMillan in Ref. 4 was able to provide this action for the condition that the power supply interval resistance,  $R_{\rm S}$  is zero value. When  $R_{\rm S}$  is not zero, additional programming is required to perform the required trigger action.

The Pulse Width Modulation (PWM) used by Askinas in Ref. 8 must be added to the present model, however Askinas used a separate additional transistor to perform PWM. The Power Conditioner to be used in the Test Stand for measurement of actuator performance accomplishes PWM by using the lower set of the commutating power transistors. Therefore some modification of Askinas' CSMP coding may be required to accurately perform the PWM speed control.

Extensions of the model will include a position sensing device and a tachometer as well as the nonlinear loading as developed by Franklin in Ref 9. Test data results will be compared to model in both transient (step) response and in frequency (Bode) response and the results will be reported in a Summary Report for FY1986.

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- 9. Franklin, G. C., "Computer Simulation of a Cruise Missile Using Brushless DC Motor Fin Control", M.S. Thesis, Department of Electrical and Computer Engineering, Naval Postgraduate School, Monterey, CA, March 1985.

the rate of change and set it equal to zero. Thus,

$$\frac{dP_{0}}{dT_{L}} = -\frac{k_{t}k_{w1}v_{sg}}{R_{A} + R_{q}} + \frac{2k_{t}k_{b}k_{w1}w_{n1}}{R_{A} + R_{q}} - \frac{2k_{t}k_{b}k_{w1}^{2}}{R_{A} + R_{q}} T_{L} = 0$$

Solving for load torque, TL yields

$$T_L = w_{nl}/k_{wl} - v_{sg}/(2k_gk_{wl})$$

$$T_{L} = (2k_{b}w_{n1} - v_{sg})/(2k_{b}k_{w1})$$

and substitution into the power output equation produces the conditions for peak power output as follows:

$$P_{o} = k_{t}v_{sg}/(R_{a}+R_{q})[w_{n1}-k_{w1}(2k_{b}w_{n1}-v_{sg})/(2k_{b}k_{w1})]$$

$$- k_{t}k_{b}/(R_{a}+R_{q})[w_{n1}-k_{w1}(2k_{b}w_{n1}-v_{sg})/2k_{b}k_{w1})]^{2}$$

After expansion and cancellation of like terms, the peak power output reduces to

$$P_o = k_t v_{sg}^2 / [4k_b(R_a + R_q)]$$

As a numerical example consider the following values of a typical Brushless DC Motor operating at a steady-state speed of 3060 RPM (320.4 rad/sec) with a constant terminal voltage of 30 volts.

 $k_t = 15.9$  oz-in/ampere

 $k_b = 0.112 \text{ volts/rad per sec}$ 

 $k_{adj} = 0.825$ 

 $R_a = 1.37$  ohms,  $R_q = 0.10$  ohms

 $k_{wl} = 1.67$  rad per sec/oz-in

Thus for this example motor,

$$P_o = (15.9)(0.825)(30)^2(7.062x10^{-3})/[4(.112)(0.825)(1.47)]$$

 $P_0 = 153.5$  watts

and occurs at load torque

$$T_{L} = [(320.4)/(1.67) - (30.)/(2)(.112)(.825)(1.67)]$$

$$T_{1.} = 94.65 \text{ oz-in} = 5.92 \text{ lb-in}$$

#### APPENDIX A

### Power Output of a Brushless DC Motor

The mechanical power developed by a Brushless DC Motor is given by the equation,

$$P = (7.062x10^{-3}) \text{ Tw}_m \text{ watts}$$

where  $T=T_L+T_M$ , the sum of the load torque,  $T_L$  and the motor restraining torque,  $T_M$  in oz-in units and  $w_m$  is the motor speed in rad/sec. units. At no load  $T_L=0$  and at stall torque  $w_m=0$ , thus power reaches a peak value in the region between no load and stall torque conditions.

The evaluation of steady-state peak power can be obtained analytically in terms of the load torque by substitution as shown below.

Given that  $T=T_L$  that is  $T_L>>T_M$  and  $T=k_{tim}$  with  $w_M=w_{NL}$  where  $k_t$  is the motor torque coefficient and  $k_{wl}$  is the slope of the speed-torque curve.

Then the power output is

$$P_0 = k_t i_m (w - k_w T_L)$$

and since steady-state speed conditions are assumed, the current is given by

$$i_{m} = \frac{v_{sg} - k_{b}w_{m}}{R_{A} + R_{G}}$$

where  $\mathbf{v}_{sg}$  is the voltage at the terminals of the power supply,  $\mathbf{k}_b$  is the Back EMF coefficient,  $\mathbf{R}_a$  is the armature resistance of 2 phases in series and  $\mathbf{R}_q$  is the sum of 2 power transistors forward conduction resistance. By substitution.

$$P_{O} = \frac{k_{t} v_{sg}}{R_{a} + R_{q}} \left( w_{nl} - k_{wl} T_{L} \right) - \frac{k_{t} k_{b}}{R_{A} + R_{q}} \left( w_{nl} - k_{wl} T_{L} \right)^{2}$$

and to find the conditions for peak power output, it is necessary to compute

The curve of power output versus load torque for this example motor is shown in Figure 9. It must be noted that the above analysis is valid for the assumption that motor current produced fields do not have a measurable effect on the magnitude and phase of the flux generated by the rotating permanent magnets and also that a constant average flux exists in the air-gap between rotor and stator.

## APPENDIX B

### CSMP Listing

The system described by the schematic diagram in Figure 4 is programmed in the CSMP language as shown below. The computer used was an IBM 370 located in the W. R. Church Computer Center at the Naval Postgraduate School. Following the program listing is a User's Guide for the recommended procedure in setting the constants and parameters for validation with measured motor performance. Also included are typical output waveforms for a step input of supply voltage.

//JRBTROO1 JCB (C169.C323) "MACHILLA-12", CIASS=G, MSGLEVEL=(1,1)
//\*MAIN CSG=N23VM1.G169F, LIN=C=999
//\*PORMAI FR, DENAME=, DESI=ICCAL
//\*PORMAI FR, DENAME=SYSVECTA, LESI=ICCAL
//CSMPSTEP EXEC CSMPVDV, PCF5IRS=MYCIFS
//X.SYSIN CC \* PATCHES FBCM 10C INSTALLED

VERSIGN PLEVEN -- SIXTH REVISION INCORFCEATING THESVS LIII.

THIS VERSION INCORFCEATES REVERSING COMMUTATION AT REVIIM.

CURRENTS ARE NOT SUPEFFECSED. FLUX \* CURBENT IS COMPUTED FOR FACH FEASE

AND THE RESULTING TOFCUES AFE SUMMED.

THE PURPOSE OF VERSION FIVE IS 10 THEAT THE FLUX AS VARYING

ACCORDING TO THE SUM OF A FUNDAPENTAL SINUSCID

AND ITS FIFTH HARMONIC AS EXPLAINED IN CHAFTER FIVE. THECHAS)

THE TOTAL FLUX IS AFPECXIMATED AS THE ALGEFRAIC SUM OF THE FLUX

DEVELOPED IN TWO WINDINGS AT A TIME IC CALCULATE THE TORQUE

COEFFICIENT AND WINDINGS AT A TIME IC CALCULATE THE TORQUE

COEFFICIENT AND WINDINGS AT A TIME IC CALCULATE THE SWITCHING

HARMONIC AND SIMULATES MAGNETIC FLUX AS A SINUSCID AND A FIFTE

HARMONIC AND SIMULATES DIODE COMMUTATION AS WELL AS THE SWITCHING

CONDITIONER. THE WINDINGS AFE NOT TREATED TO RETURN FOWER

CONDITIONER. THE WINDINGS AFE NOT TREATED TO SIMULATED,

THE CONTRIBUTIONS TO DEVELOPED TOFQUE ARE

THEATED AS SUFERFOSABLE. THE PROGRAM HAS EEEN MCDIFIED TO INCLUDE FOWER SUFFLY RESISTANCE, BS. WITH THE ADDITION OF BS. THE IRIGGERING VOITAGE FOR THE TRANSISTOR PROTECTING DIODES HAS EEEN CHANGED TO INSURE PROPER CEFFATION. INITIAL CONSTANT KTM = 15.86, EX = 0.01465, EI = 0.00, JI = 0.0, N = 1.0, ... JH = 0.001, KEM = 0.1120, EI = 3.14159265, KR3 = 3.590, KACJ = 63, ... THADV= 0.0, ES= 8.33, ICINC = 0.01 PARAMETER LA = .COO8, FA = 0.685C, TLI= C.C PARAMETER VSAT = .4, RSAT= .C5, RCET = 1.CE+4 PARAMETER TITIME = 1.0E-6, REVIIM = C.10 PARAMETER VC1D=0., VC2C=C., VD3D=0., VC4C=0., VC5C=C., VC6C=C. PARAMETER VSGPEF = 3C.C, KINT = 1CC0C.0, KRAME = 18250.C RTM -- MFG SUPPLIED MCTOR TORQUE CONSTANT (C2-IN/AMP)

RT -- FHASE ICACUE CONSTANT (C2-IN/AMP)

RBM -- AFG SUPPLIED MCTOR TORQUE CONSTANT (VCIT/RAD/SEC)

RBM -- AFG SUPPLIED MCTOR FCR MFG SUPPLIED MCTOF CONSTANTS

RA -- EHASE BACK EMF CONSTANT (VCIT/RAD/S)

RA -- EHASE BACK EMF CONSTANT (VCIT/RAD/S)

RA -- PHASE INDUCTANCE OF THE MCTOR (FENRIES)

BA -- VISCOUS FRICTION COFFFICIENT OF THE MCTOB (CZ-IN/RAD/S)

BL -- VISCOUS FRICTION COFFFICIENT OF THE LOAL (CZ-IN/RAD/S)

BL -- VISCOUS FRICTION COFFFICIENT OF THE MCTOB (CZ-IN/RAD/S)

BL -- TOTAL VISCOUS FRICTION COFFFICIENT OF THE MCTOB (CZ-IN/RAD/S)

JH -- INEFTIA OF THE MCTOR (CZ-IN/S-S)

JL -- INEFTIA OF THE MCTOR (CZ-IN/S-S)

JL -- INEFTIA OF THE MCTOR SYSTEM (SFE FEICH, NC-SOFT SECTION)

A2 = J/E -- THE MCCAN THEU BELUCTION GEARS

J -- TOTAL INERTIA OF THE MCTOR SYSTEM (SFE FICH, NC-SOFT SECTION)

A2 = J/E -- THE MCCAN THE LOAD THEE ELECTRICAL TIME CONSTANT OF THE MCTOR

AND LAIVA TANNSTOCKS FOR CURFERN FAIH A-E

BCTAU = LA/(SA+BECAE) -- THE ELECTRICAL TIME CONSTANT OF THE MCTOR

AND LAIVA TANNSTOCKS FOR CURFERN FAIH A-E

BCTAU = LA/(SA+RECICA) -+ THE ELECTRICAL TIME CONSTANT OF THE MCTOR

AND DRIVE TRANSISTORS FOR CURFERN FAIH A-E

CATAU = LA/(SA+RECICA) -+ THE ELECTRICAL TIME CONSTANT OF THE MCTOR

AND DRIVE TRANSISTORS FOR CURFERN FAIH CAND CONSTANT OF THE MCTOR

AND DRIVE TRANSISTORS FOR CURFERN FAIH CAND CONSTANT OF THE MCTOR

AND DRIVE TRANSISTORS FOR CURFERN FAIH CAND CONSTANT OF THE MCTOR

AND DRIVE TRANSISTORS FOR CURFERN FAIH CAND CONSTANT OF THE MCTOR

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AND DRIVE TRANSISTORS FOR CURFERN FAIH CAND CONSTANT OF THE MCTOR

AND DRIVE TRANSISTORS FOR CURFERN FAIH CAND CUTCEF

CATAU = LA/(SA+RECICA) -+ THE ELECTRICAL TIME CONSTANT OF THE MCTOR

AND DRIVE TRANSISTORS FOR CURFERN FAIH CAND CONSTANT OF THE MCTOR

AND DRIVE TRANSISTORS FOR CURFERN FAIH CAND CONST

```
CSME
                                                                                              A 1
FILE: BHIRA
       RSAT = EQUIVALENT CLICUIT FESISTANCE OF TRANSISTOR SATURATION ROUT = EQUIVALENT CLICUIT FESISTANCE OF TRANSISTOR CUICFF TIME -- SWITCHING TIME OF TRANSISTOR THEST -- THIS IS THOSE WHICH IS SET TO ZERO AT STABL, RESEL TO ZERO AT 300 DEG AND CAN ALSO SE USEL AS SIMULATION SICE ANGLE. THADY -- THETA ADVANCE FOR COMMULATION (COM IS POSITIVE) THOON -- COMMUTATION CONTROL ANGLE THESE +THALY IN DEGREES RS --- INTERNAL BESISTANCE OF THE SUPPLY VOLTAGE
 NOSCRI
        JSCRT

RT=RTd+KACJ

RB=RBM+KACJ

BLP = EL/(N**2)

JLP = JL/(N**2)

J = JM + JLP

E = EM + ELP

A1 = IA / EA

A2 = J / E

A3 = LA/(EA +RSAI)

RS1=ES/2.C
        RS1=RS/2.0
RS2=RS/2.0
VINIC = 15.0/KINT
 METHOD SILEF
DYNAMIC
VIF VINHAF
        VIB = VINHAF
VIN = VIF - VIE
       VIN = VIF - VIE

VSGDEL = (VSGREF - VSG)/2.0

VSGEER = DEADS?(-0.5. +0.5. V

VININT = INTGRI (VINIC, VCGEAS)

VINHEG = KINI + VINIT

VINHAP = VINEF1 - VINEF2

VRAMP1 = BAMP(0.001)

VRAME2 = BAMP(0.001)

VINHET = KRAME+VEAME2
                                                                                                                        VSGLEL)
       INTRODUCE FINITE IBANSILION TIME FOR IBANSISION SWITCH
BY INCOPPORATING EXECNERITAL RISE AND DECAY INTO SWITTEN
OFEXP PROVIDES A REALISTIC EXPONENTIAL IFANSITION EFTREEN CUTOFF
AND SATURATION WHICH INCLUDES SHIDFATION DELAY WHEN PUT THEOUGH
THE LIMITED
Q4EXF=0.1, Q5EXE=0.1, Q6EXF=0.1
Q4EXF=0.1, Q5EXF=0.1, Q6EXF=0.1
```

+ ( ( 1 ) + ( ( 1 ) + ( ( 1 )

VCIND VAIND VAIND VAIND VAIND

LINEAR CICDE MODEL
DIODE 108A-CN VCLIAGES
VD1D = 6000.0 - VAIND +
VD6D = 6000.0 - VCIND VD3D = 6000.0 + VCIND +
VD2D = 6000.0 + VAIRD VD5D = 6000.0 + VB5D VD5D = 6000.0 + VB5D

84

A 1

```
ENDFRCCEDURE
```

lease research seeded about the

```
THIS PAGCEDURE SIMULATES GENERAL COMMUTATION,
DETERMINES THE ALGEBRAIC SUM OF
THE VARIABLE FLUX COMPONENTS FOR USE IN COMPUTING THE MOTOR'S
APPROXIMATE BACK EMF (BEMFI). IT ALSO SUBTRACTS THE GENERATED
VOLTAGE FROM THE PROPER SUPPLY VOLTAGE.
SWI THRU SW6
SIMULATE POWER TRANSISTOR TRIGGERS BEING ENERGIZED OR SWITCHED OFF
THOON IS THE VARIABLE THROUGH WHICH THE SWITCHING LOGIC
IS IMPLEMENTED. REVTIM IS THE TIME AT WHICE CLOCKWISE COMMUTATION
BEGINS.
# BEGINS.

PROCEDURE Sh1, SW2, SW3, SW4, SW5, SW6, EEPFT, VN1, VN2 = CCMM(TIPE, ...)

REVIIM, THCCN, BEMFA, EEBFE, LEMFC, VEMFA, VEMFE, VEMFC)

IF (THECN1 = ANDL (THCCN, 180.)

IF (THCON1 = IT CON) = THCON1 = THCCN1 + 180.

IF (THCON1 GE. JO. AND. THCON1 = IT GC.) GC IC 50

IF (THCON1 GE. JO. AND. THCON1 LIT. 3C.) GC IC 51

IF (THCON1 GE. 9C. AND. THCON1 LIT. 9C.) GC IC 52

IF (THCON1 GE. 9C. AND. THCON1 LIT. 180.) GC IC 52

IF (THCON1 GE. 150. AND. THCON1 LIT. 180.) GC IC 55

**CLOCKWISE COMMUTATION

IF (THCCN1 GE. JO. AND. THCON1 LIT. 30.) GC IC 52

IF (THCCN1 GE. JO. AND. THCON1 LIT. 30.) GC IC 52

IF (THCON1 GE. JO. AND. THCON1 LIT. 30.) GC IC 52

IF (THCCN1 GE. JO. AND. THCON1 LIT. 30.) GC IC 52

IF (THCON1 GE. JO. AND. THCON1 LIT. 30.) GC IC 52

IF (THCCN1 GE. JO. AND. THCON1 LIT. 30.) GC IC 52

IF (THCCN1 GE. JO. AND. THCON1 LIT. 30.) GC IC 52

IF (THCCN1 GE. JO. AND. THCON1 LIT. 30.) GC IC 52

IF (THCCN1 GE. THCON1 LIT. 180.) GC IC 52

IF (THCCN1 GE. THCON1 LIT. 180.) GC IC 52

SW1 = C.

SW2 = O.

SW3 = G.

SW3 = G.

SW3 = I.

SW2 = O.

SW3 = G.

SW3 = J.
                                     900100
             53
                                         SN6 = 1.
BEMFT = EEMFE - BEMEC
```

```
A 1
                                                                    CSMP
 FILE: EATEA
    VN1 = VIF - VEMPE

VN2 = VIE - VEMPC

IN=INB

GC IC 60

SW1 = C.

SW2 = 1.

SW3 = 1.

SW4 = 0.

SW5 = C.

SW6 = C.

BEMPE - LEMPA

VN1 = VIF - VEMPA

VN2 = VIE - VEMPA

IM=IME
VN 2 = VIB - VEMFX

IM = IME

GO TO CO

55 SW 1 = 0.

SW 2 = 1.

SW 3 = 0.

SW 4 = 0.

SW 6 = 0.

BENPT = EEMFC - EPMFA

VN 1 = VIF - VEMFC

VN 2 = VIB - VEMFA

CONTINUE

ENDPROCECUBE
TERMINAL

TITLE BASIC DC MOICE SYSTEM

TIMES FINITM = .050, CUTDEL = .0000275, FREEL = .000275,...

DELMIN=1.0E-10

PINISH THASI = 400.0

PRINT MA, WAREN, THRST, IM, TMINI, IMINI, WMINI, IAV, TAV, WMAV, VAIND, VEIND, VCIND, VN1, VN2, VAC, VEC, VCC, C3, VCD4, VCD5, VCC6, VSG, VAG, VEG, VCGC, VD1D, VD2D, VD3E, VCC4, VCCC, C3, VCD4, VCD5, VCC6, IAB, IABA, IACA, ICAA, ICAA, IM, IEEEG, THCCN, THCON, IMA, IMB, IMC, FAF, VIN, VINHAF, IMCAC, VSGF, TMM, TEM, VKEWM

LABEL EHASE C CURRENT - AMES

OUTPUT TIME, IMC, SW2

FAGE XYELCT

END

RESET FRINT

LABEL FOWER SUPPLY CURPENT TO MCICE - AMES

CUTFUT TIME, IM, SW4

PAGE XYELCT

END

LABEL ACICE SEEEL - SEE
         LABEL MOTOR SPEED - REM
OUTPUT TIME, WHAEM, SAS
PAGE XYFLOT
END
LABEL FOWER SUFFLY TERMINAL VOITAGE - VOLTS
CUTPUT TIME, VSG, SAC
PAGE XYFLOT
                                END
         LABEL TOTAL DEVELOPED TORQUE - CZ-IN OUTPUT TIME, TH,SW1
PAGE YYELCT
         LABEL EF-FILTERED PHASE C CURRENT - AMPS
CUTPUT TIME, INCAC, SW3
PAGE XYFICI
END
LABEL VOLTAGE ACEOSS A-C TEPMINALS - VOLTS
OUTPUT TIME, VAC, SW3
PAGE XYFICI
                                 END
         LABEL "IP-FILIERED POWER SUPPLY CURRENT -AMPS CUIPUT TIME, IMF, SK5 PAGE XYELCT"
                                 FVC
          LABEL BACK EMF PHASE C - VOLTS
```

```
OUTPUT TIME, VEMFC, SW1
PAGE XYFICT

LABEL IN CLYLICFED ICFQUE - CZ-IN
OUTPUT TIME, THM, SW3
PAGE XYFICT

LABEL FCWER OUTCUT - WATTS
OUTPUT TIME, PNR, SW5
PAGE XYFICT

LABEL IP-FILTERED SUPPLY TERMINAL VOLTAGE - VOLTS
OUTPUT TIME, VSGF, SW1
PAGE XYFICT

END
STCP
ENDJOB

/*DISSECF.MYDIRS DD *
DRAW=1-END
MODIFY=1-END(3) (SIZE=8.,3.), NEE=0.,3., SIZE=8.,3.),
MODIFY=2-END(3) (GVEFFLOT, CCRNEF=0.,3., SIZE=8.,3.),
MODIFY=3-END(3) (GVEFFLOT, CCRNEF=0.,3., SIZE=8.,3.),
```

THE PROPERTY OF THE PARTY OF TH

#### Users Guide

The following procedures are recommended to be used in running the CSMP language program given in Appendix B:

- Obtain the motor, load and power supply/conditioner data from the manufacturer and enter the values in INITIAL section at the beginning of the program.
- 2. Set parameter KRAMP to the desired input voltage V as follows: example V = 30 volts, KRAMP =  $\frac{V}{2} \times 10^3$  = 15000. Under these conditions, a fast terminated-ramp input results. This is equivalent to a step input of V volts.
- 3. The constant KADJ must be set experimentally to produce the measured motor Back EMF voltage. To do this, remove the asterisk in the statement

\* WM = 320.4475

where 320.4475 is the magnitude of the driver motor speed in rad/sec. (in this case, speed = 3060 RPM). Run the program with different values of KADJ until the Back EMF across 2 windings agree in magnitude with the measured value obtained from the motor.

- 4. Set N to the value of the speed reduction from motor shaft to load shaft.
- 5. Set RS to the value of the power supply interval resistance and LA and RA to the per phase winding values.
- 6. Set RSAT and VSAT values for the power transistor "ON" values and RCUT to the cut-off resistance value.

7. The value of BM should be calculated from the no-load data as a preliminary value and adjusted experimentally until no-load operation of the model agrees with measured no-load values for the motor. Compute BM as follows:

BM = (KT)(KADJ)(IMNL)/(WMNL)

For example

BM = (15.89)(.63)(0.3)/(320.4)

BM = 0.00937 oz-in/rad per sec.

Notes: 1. The asterisks in column 1 of the program statement makes the statement a comment and disregarded by the CSMP Translator. These statements are the result of program development and are included for future application in the development of the complete actuator model.

- 2. A set of typical output waveforms are included here to indicate the plot output capabilities of the program which are in the TERMINAL section of the CSMP listing.
- 3. The load applied to the motor should be adjusted over the range of no-load to near peak load and the performance curves of Figure 12 plotted. The CSMP simulation output can then be checked against these results by entering BL as follows:

BL = TL/WM oz-in/rad per sec where TL is the load torque (oz-in) WM is the motor speed (rad/sec).

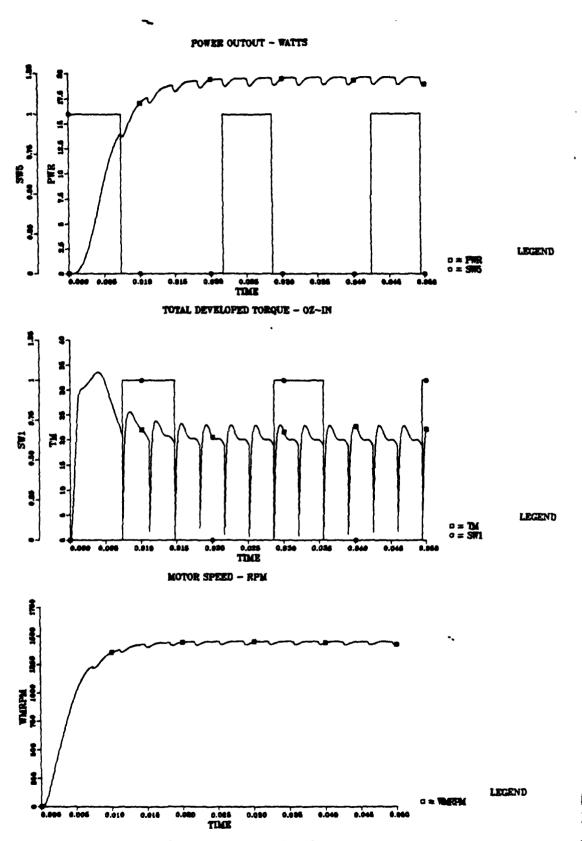
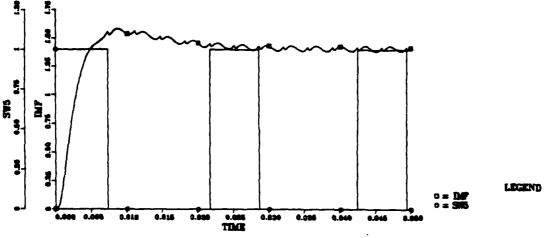
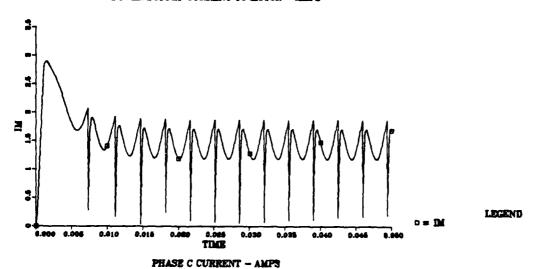


Figure B1 Step Input Response Waveforms

# LP-FILTERED POWER SUPPLY CURRENT -AMPS



POWER SUPPLY CURRENT TO MOTOR - AMPS



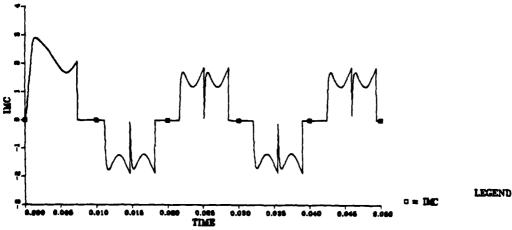


Figure B2 Additional Step Response Waveforms

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